Introduction to the Theory of Computing 2. 5/29/2019

Test for the Signature

1. How many six-digit integers are there whose digits are all different and the even and odd digits alternate? (E.g. 123456 and 250143 are such integers.)

2. In a simple graph with two components exactly four vertices have odd degrees. Moreover we know that neither of the components comtain an Euler circuit. Is it always true that we can add two edges to the graph in such a way that the graph obtained is still simple and contains an Euler circuit?

3. In a simple bipartite graph on 20 vertices the degree of each vertex is at least 9. Show that this doesn't imply that the graph contains a Hamilton path.

4. Let the vertices of the graph G be the numbers $1, 2, \ldots, 100$, and two (different) vertices be adjacent if and only if at least one of 2, 3 or 5 is a common divisor of the respective numbers. Determine $\chi(G)$, the chromatic number of the graph G.

5. Let the two vertex classes of the bipartite graph $G(A, B; E)$ be $A = \{a_1, a_2, \ldots, a_9\}$ and $B = \{b_1, b_2, \ldots, b_9\}$. For each $1 \le i \le 9$ and $1 \le j \le 9$ let a_i and b_j be adjacent if the entry in the ith row and jth column of the matrix below is 1. Determine a maximum matching and a minimum covering set of vertices in G.

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

6. $(*)$ We obtained the graph G from K_9, the complete graph on 9 vertices by deleting 4 edges from it. Is it possible that the edge-chromatic number of G is 8?

Total work time: 90 min.

The full solution of each problem (including explanations) is worth 10 points. Show all your

work! Results without proper justification or work shown deserve no credit.

Notes and calculators (and similar devices) cannot be used.