Introduction to Computer Science

Repeated Second Midterm Test, 5/9/2016

1. Let the vertices of the graph G be the all the 0-1 sequences of length 5, and two sequences be adjacent if they differ in eactly one position. Is the graph G a bipartite graph?

2. Let the vertex set of the graph G be $V(G) = \{1, 2, \ldots, 30\}$. Let the vertices $x, y \in V(G)$ be adjacent in G if the difference of the numbers x and y is at least 7. Determine $\chi(G)$, the chromatic number of G.

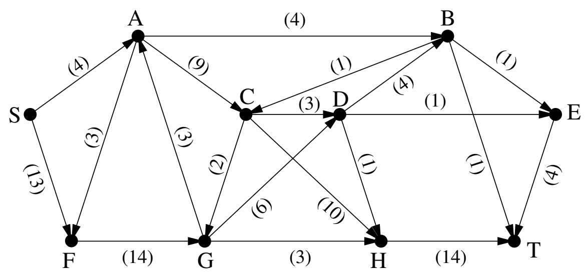
3. Let the two vertex classes of the bipartite graph $G(A, B; E)$ be $A = \{a_1, a_2, \ldots, a_8\}$ and $B = \{b_1, b_2, \ldots, b_8\}$. For each $1 \leq i, j \leq 8$ let a_i and b_j be adjacent if the entry in the ith row and jth column of the matrix below is 1. Determine $\nu(G)$, the maximum number of independent edges, $\rho(G)$, the minimum number of covering edges, and give a maximum matching and a minimum covering set of edges in G.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

4. The chromatic number of the simple graph G is $\chi(G) = 3$ and there is a coloring of the vertices of G with 3 colors in which one of the colors appears on one vertex only. Show that $\tau(G) \leq \nu(G) + 1$ holds for G, where $\tau(G)$ is the minimum number of covering vertices and $\nu(G)$ is the maximum number of independent edges in G.

5. Let the vertex set of the graph G be $V(G) = \{1, 2, \ldots, 30\}$. Let the vertices $x, y \in V(G)$ be adjacent in G if $x \neq y$ and $x \cdot y$ is divisible by 7. Determine $\chi_e(G)$, the edge-chromatic number of G.

6. Determine a maximum flow and a minimum cut in the network below.



Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points.
Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 51-60 points: 5.