Second Repeat of the Second Midterm Test

1. Is there a simple bipartite graph on at least 5 vertices whose complement is also a bipartite graph?

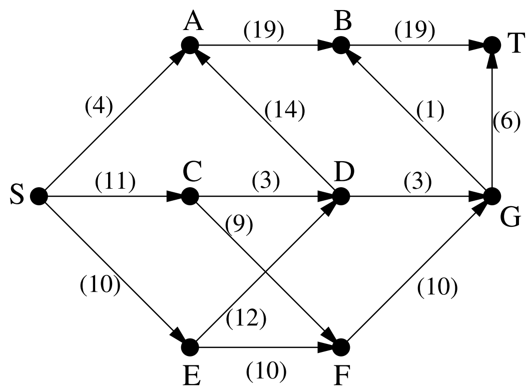
2. The graph G on 16 vertices consists of 2 vertex-disjoint cycles on 8 vertices each. Determine $\chi(\overline{G})$, the chromatic number of the complement of G.

3. Somebody placed coins on some of the squares of the (8×8) chessboard. We don't know the weights of the individual coins (they are not necessarily the same), but we know that the total weight in each row and column is 12 grams. Show that we can select 8 coins in such a way that each row and column contains exactly one of them.

4. We call vertex v of the graph G *important*, if after deleting v (and the edges incident to it) from G, for the graph G' obtained $\alpha(G') < \alpha(G)$ holds. At most how many important vertices can a graph on 100 vertices have if $\alpha(G) = 10$? ($\alpha(G)$ is the maximum number of independent vertices in G.)

5. We substitute each edge of K_4, the complete graph on 4 vertices by 2 parallel edges, then subdivide one of the edges of the graph obtained. Determine $\chi_e(G)$, the edge-chromatic number of the final graph G (with 5 vertices and 13 edges). (The subdivision of an edge means that we substitute the edge $\{u, v\}$ by the edges $\{u, x\}$ and $\{x, v\}$, where x is a new vertex.)

6. Determine a maximum flow from S to T in the network below.



Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points.
Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 51-60 points: 5.