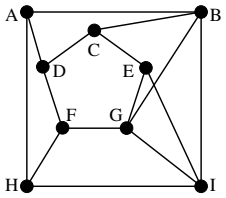
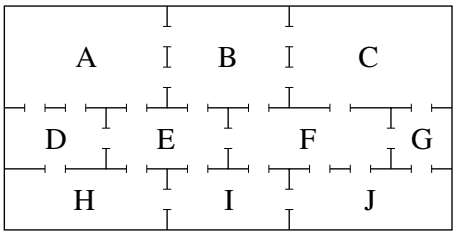
1. The neptun code of a student is a sequence consisting of 6 characters, each of which is either one of the 26 letters of the English alphabet or one of the digits $0, 1, \ldots, 9$. How many neptun codes are there which contain at most two digits?

2. The graph G on 6 vertices can contain multiple edges, but no loops. We know that the degree of any two vertices of G are different. At least how many edges are there in G? (That is, for which integer k does it hold that there is a graph with this property with k edges, but not with less than k edges?)

3. Decide whether the following graph is planar or not. If yes, then draw it with straight edges without crossing, if not, then prove it.



4. The picture below is the layout of a royal palace. Each morning the king starts a walk from his apartment denoted by A. He decides that he wants to pass through each door exactly once. If he succeeds in it, then he would assign the last room of the walk as the throne-room. But he never succeeds in it, so the court sage tells him to wall up one of the doors. Is there a door in the palace, such that after walling it up there is a walk satisfying the king's wish? If yes, which room can become the throne-room?



5. In the simple planar graph G the degree of each vertex is at least 4 and the number of vertices of degree exactly 4 is five. Show that G doesn't contain an Euler circuit.

6. In the simple graph G on 201 vertices the degree of each vertex, except for v, is at least 101. About v we only know that it is not an isolated vertex. Show that G contains a Hamilton path.

Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points.
Grading: 0-24 points: 1, 25-33 points: 2, 34-42 points: 3, 43-51 points: 4, 52-60 points: 5.