Second Midterm Test

1. White and black knights are standing on a chessboard, altogether 7 of them. Each of them attacks at least two knights of the other color. Show that all the white knights are standing on squares of the same color. (A knight in one move goes two squares vertically or horizontally and then one more square perpendicularly to the previous direction.)

2. In a tree on 11 vertices each vertex has degree at most 3. Show that the tree has a matching of 4 edges.

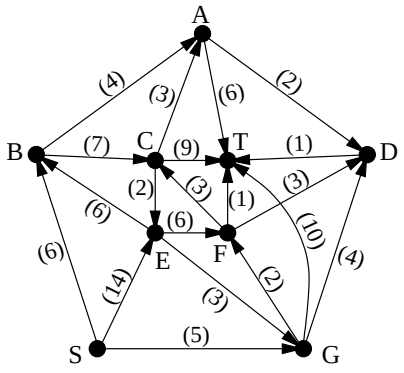
3. We delete the edges of a Hamilton cycle from a complete graph on 100 vertices. Determine the chromatic number of the graph obtained.

4. Determine whether the following graphs are interval graphs or not.

5. Let the two vertex classes of the bipartite graph $G(A, B; E)$ be $A = \{a_1, a_2, \ldots, a_9\}$ and $B = \{b_1, b_2, \ldots, b_9\}$. For each $1 \leq i \leq 9$ and $1 \leq j \leq 9$ let a_i and b_j be adjacent if the entry in the ith row and jth column of the matrix below is 1. Determine a maximum matching and a minimum covering set in G.

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

6. Determine a maximum flow and a minimum cut in the network below.



Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points.
Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 51-60 points: 5.