1. Determine the chromatic number of the graph below.

2. Determine the maximum number of independent edges in the graph above.
3. Let $G$ be a simple graph on 7 vertices for which both $G$ and its complement $\bar{G}$ contain a Hamilton-cycle. Prove that the chromatic number of $G$ is either 3 or 4 .
4. Let $S=\left\{a_{1}, a_{2}, \ldots, a_{n}\right\}$ be an $n$-element set, and let $S_{1}, S_{2}, \ldots, S_{m}$ be subsets of $S$. Suppose that each of the subsets contain at least 4 elements, and that each element is contained in at most 3 subsets. Prove that we can select $m$ distinct elements, one from each of the subsets.
5. Show that if $G$ is a simple $k$-regular graph on 9 vertices then $\chi(G)+\chi(\bar{G}) \geq 10$.
6. Determine a maximum flow and a minimum cut in the network below.


Total work time: 90 min .
The full solution of each problem (including explanations) is worth 10 points.
Grading: $0-23$ points: $1,24-32$ points: 2 , $33-41$ points: 3 , $42-50$ points: $4,51-60$ points: 5 .

