Introduction to Computer Science Repeated Second Midterm Test, 5/4/2014

1. Determine the chromatic number of the graph below.



- 2. Determine the maximum number of independent edges in the graph above.
- 3. Let G be a simple graph on 7 vertices for which both G and its complement \overline{G} contain a Hamilton-cycle. Prove that the chromatic number of G is either 3 or 4.
- 4. Let $S = \{a_1, a_2, \ldots, a_n\}$ be an *n*-element set, and let S_1, S_2, \ldots, S_m be subsets of *S*. Suppose that each of the subsets contain at least 4 elements, and that each element is contained in at most 3 subsets. Prove that we can select *m* distinct elements, one from each of the subsets.
- 5. Show that if G is a simple k-regular graph on 9 vertices then $\chi(G) + \chi(\overline{G}) \ge 10$.
- 6. Determine a maximum flow and a minimum cut in the network below.



Total work time: 90 min.

The full solution of each problem (including explanations) is worth 10 points. Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 51-60 points: 5.