## Second Repeat of the Second Midterm Test

1. In a simple graph $G$ on 10 vertices the degree of each vertex is 8 . Determine the chromatic number of $G$.
2. In a simple graph $G$ on 10 vertices the maximum degree is 6 , and the maximum number of independent edges is 5 . Show that $G$ contains an odd cycle.
3. In a simple bipartite graph $G$ on 9 vertices the degree of each vertex is either 2 or 4. Show that $G$ contains a matching of 3 edges.
4. A simple graph $G$ on 10 vertices contains one vertex of degree 5 , one of degree 4 , one of degree 3 , and the rest of the vertices have degree 2 . Show that $G$ can be colored with 3 colors.
5. Show that if $G$ is a simple $k$-regular graph on 9 vertices then $\chi_{e}(G)+\chi_{e}(\bar{G}) \geq 10$.
6. Determine a maximum flow and a minimum cut in the network below.


Total work time: 90 min.
The full solution of each problem (including explanations) is worth 10 points.
Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, $42-50$ points: 4, $51-60$ points: 5 .

