Second Repeat of the First Midterm Test

1.

- 2. How many sequences of length 4 are there which contain 4 different numbers from 1, 2, ..., 100 such that the largest number is the first one and the others are in increasing order? (E.g. 25,5,10,16 is such a sequence.)
- 3. Let G be a simple graph and $v \in V(G)$ be a vertex of odd degree. Show that there is a path in G whose starting vertex is v and end vertex in a vertex of odd degree different from v.
- 4. The graph G doesn't contain a subgraph homeomorphic to $K_{2,3}$ (the complete bipartite graph on 2+3 vertices). Does it follow that G is planar?
- 5. Does the graph below conatin a
 - a) Hamilton path,
 - b) Hamilton cycle?



6. 11 children play a game. THey stand in a circle, and one of them starts passing a ball to somebody else, who in turn passes it on to a third child, etc. The rules are: nobody can throw the ball to somebody he/she has thrown it before, also nobody can throw the ball to somebody who has thrown it to him/her before, and nobody can throw the ball to either of the 2 children standing next to him/her in the circle. At most how many passes are possible in the game with these rules?

Total work time: 90 min.

The full solution of each problem (including explanations) is worth 10 points. Grading: 0-23 points: 1, 24-32 points: 2, 33-41 points: 3, 42-50 points: 4, 51-60 points: 5.