## Second Repeat of the First Midterm Test

1. 
2. How many sequences of length 4 are there which contain 4 different numbers from $1,2, \ldots, 100$ such that the largest number is the first one and the others are in increasing order? (E.g. $25,5,10,16$ is such a sequence.)
3. Let $G$ be a simple graph and $v \in V(G)$ be a vertex of odd degree. Show that there is a path in $G$ whose starting vertex is $v$ and end vertex in a vertex of odd degree different from $v$.
4. The graph $G$ doesn't contain a subgraph homeomorphic to $K_{2,3}$ (the complete bipartite graph on $2+3$ vertices). Does it follow that $G$ is planar?
5. Does the graph below conatin a
a) Hamilton path,
b) Hamilton cycle?

6. 11 children play a game. THey stand in a circle, and one of them starts passing a ball to somebody else, who in turn passes it on to a third child, etc. The rules are: nobody can throw the ball to somebody he/she has thrown it before, also nobody can throw the ball to somebody who has thrown it to him/her before, and nobody can throw the ball to either of the 2 children standing next to him/her in the circle. At most how many passes are possible in the game with these rules?

Total work time: 90 min .
The full solution of each problem (including explanations) is worth 10 points.
Grading: $0-23$ points: $1,24-32$ points: 2 , $33-41$ points: $3,42-50$ points: $4,51-60$ points: 5 .

