

Intro to CS
Test #2
December 11, 2006

1. Assume that vector v is an eigenvector of matrix A . What is the value of p if v and A are the following? Compute an eigenvalue of A .

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 6 & 6 \\ 1 & -2 & -1 \end{pmatrix}, \quad v = \begin{pmatrix} p \\ 1 \\ -1 \end{pmatrix}$$

2. Let $V = \mathbb{R}^2$ and $\mathcal{A} : V \rightarrow V$ a linear transformation. We know that the image of vectors $b_1 = (1, 0)$ and $b_2 = (1, 1)$ are $\mathcal{A}(b_1) = 3b_1 + x \cdot b_2$ and $\mathcal{A}(b_2) = b_1 + 4b_2$ for some real number x . Determine the value of x , if for some vector $v = (y, 3)$, $\mathcal{A}(v) = v$?

3. From the complex solutions of the equation

$$(6 - i)z^5 = (3i + 1)^2 + 9$$

what is the algebraic form of that z that has the largest imaginary part.

4. Determine all complex numbers, such that $|z + i| = 1$ and $|z - i| = \sqrt{5}$.
5. Santa Clause visits a small village where there are 20 children. There are 10 types of toys for presents, from each types there are as many pieces as is necessary. In how many ways can Santa give presents to the children, if each child gets exactly 3 toys (of different types)?
6. On a 10×10 chessboard a knight is on the bottom left corner. In one step it either moves 2 squares to the right and 1 up, or 1 square to the right and 2 up. In how many ways can it reach the right corner at the top of the board?
7. In a simple graph of 100 vertices the degree of every vertex is at least 33. Show that the graph cannot have more than 2 connected components.
8. Can this graph be drawn on the plane?

