Intro to CS
Test #1
October 27, 2006

1. Decide if the point $P = (2, 7, 3)$ lies on the line connecting the two points $Q = (6, 3, 5)$ and $R = (12, -3, 8)$.

2. Assume that the vectors $v_1, v_2, \ldots, v_n$ span the vector space V. Let $u \in V$ be any nonzero vector. Show that there exists an i, such that the vectors $v_1, \ldots, v_{i-1}, u, v_{i+1}, \ldots, v_n$ also span the space V.

3. Determine all real number c, such that this system of linear equations has solution. When the system is solvable, determine all solutions.

$$\begin{array}{rcrcrcrl}
-x_1 & - & 3x_2 & + & x_3 & - & 4x_4 = & 1 \\
5x_1 & + & 15x_2 & - & 2x_3 & + & 26x_4 = & 4 \\
2x_1 & + & 6x_2 & + & & & c \cdot x_4 = & 4 \\
4x_1 & + & 12x_2 & + & x_3 & + & (c+14) \cdot x_4 = & 11
\end{array}$$

4. Let $\pi(1), \pi(2), \ldots, \pi(n)$ be a permutation of the numbers $1, 2, \ldots, n$. Let A be the following $n \times n$ matrix: in the i^{th} row of A the first $\pi(i) - 1$ entries are 0 and the others are 1 (starting from position $\pi(i)$ in the row). Compute the determinant of A.

5. Let A be a 100×100 matrix, such that for every row the sum of the entries in that row is 1. Let B denote the 100×100 matrix in which every entry is 2. Determine the matrix $A \cdot B$.

6. What is the inverse of this matrix?

$$\begin{pmatrix} 1 & 2 & 5 \\ 1 & 2 & 4 \\ 1 & 3 & 7 \end{pmatrix}$$

7. In a matrix A every row is an arithmetic progression. Show that $\mathrm{r}(A) \leq 2$. (A sequence of numbers $x_1, x_2, \ldots, x_n$ is an arithmetic progression if there exists a number t, such that $x_2 = x_1 + t$, $x_3 = x_1 + 2t$, ..., $x_n = x_1 + (n-1) \cdot t$.)

8. Let A be an $n \times n$ matrix. True or false:
(a) If $A \cdot B = \mathbf{0}$ for some $n \times n$ matrix B then $\det A = 0$.
(b) If $\det A = 0$ then there is an $n \times n$ matrix B, such that $A \cdot B = \mathbf{0}$.
(Here $\mathbf{0}$ denotes the zero matrix.)