

Intro to CS
Repeated test #2
December 15, 2006

1. We know that vector v is an eigenvector of matrix A , where

$$A = \begin{pmatrix} 1 & -1 \\ 3 & p \end{pmatrix}, \quad v = \begin{pmatrix} 1 \\ -3 \end{pmatrix}.$$

What is the value of the parameter p ? What are the eigenvalues of A ?

2. Let $V = \mathbb{R}^2$ and $\mathcal{A} : V \rightarrow V$ be a linear transformation. The image of vectors $b_1 = (1, 1)$ and $b_2 = (1, -1)$ are $\mathcal{A}(b_1) = b_1 + y \cdot b_2$ and $\mathcal{A}(b_2) = x \cdot b_1 + b_2$ for some real numbers x and y . Determine the value of x and y if we know that $(3, 1) \in \ker \mathcal{A}$.

3. What is the algebraic form of z for which

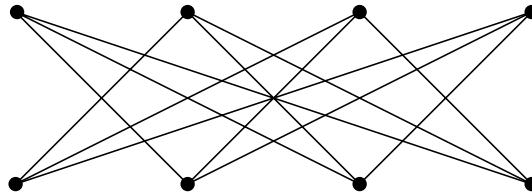
$$(2i + 3)z^3 + 3i = 2.$$

4. Determine those complex numbers z , for which there exists positive integer n , such that $z^n = 1$ and $|z + 1| = \sqrt{3}$.
5. Consider all strings of length 6 formed from the letters A,B,C,D,E,F in such a way that every letter can be used at most ones in every string. Imagine that these strings are listed in alphabetical order. What is the number of the string EABCFD in this ordering?

6. Prove the following equation:

$$\binom{100}{30} = \binom{10}{0} \binom{90}{30} + \binom{10}{1} \binom{90}{29} + \binom{10}{2} \binom{90}{28} + \cdots + \binom{10}{10} \binom{90}{20}$$

7. Let the vertices of the graph G be the 0-1 sequences of length 3. Two vertices are connected by an edge if the two sequence differs at exactly one position. Is this G isomorphic with the one on the picture?



8. Let G be a graph in which the degree of every vertex is at least 3. Assume that every cycle in G contains at most 5 edges. Show that in this case G is planar.