Intro to CS I.
Repeated Test #1
November 13, 2006

1. Decide if the line passing through the points $P = (1, 4, 4)$ and $Q = (3, 12, -2)$ has a point on any of the coordinate axes. If yes, compute all its point on the axes.

2. Let $V = \mathbb{R}$, i.e., the vectors are the real numbers. Define the addition on vectors as $u \oplus v = u + v + 1$ and the multiplication by scalar as $a \odot u = a \cdot u + a - 1$. (Here $+$ and $\cdot$ mean the usual addition and multiplication of real numbers.) Does V with operations $\oplus$ and $\odot$ form a vector space?

3. Assume that $v_1, v_2, \ldots, v_n$ are linearly independent vectors of a vector space V. Let $u \in V$ be any nonzero vector. Show that there exists an $1 \leq i \leq n$, such that the vectors $v_1, \ldots, v_{i-1}, u, v_{i+1}, \ldots, v_n$ are also linearly independent.

4. Solve the system of linear equations below.

$$\begin{array}{rcrcrcrl} 2x_1 & + & 10x_2 & + & 4x_3 & - & 6x_4 = & 2 \\ 3x_1 & + & 15x_2 & + & 11x_3 & + & 6x_4 = & 8 \\ 5x_1 & + & 25x_2 & + & 13x_3 & - & x_4 = & 8 \\ 2x_1 & + & 10x_2 & + & 6x_3 & + & 3x_4 = & 4 \end{array}$$

5. Let A be a 100×100 matrix such that $\det A = 1$. The matrix B is obtained from A in the following way: we have chosen 2 rows from A and to both row we added 7 times the difference of the two chosen rows. Determine $\det B$.

6. Let A be a 100×100 matrix, such that in the first 50 columns every entry is 3 and in the last 50 columns every entry is 2. In the 100×100 matrix B for every column it holds that the sum of the first 50 entries is 2 and the sum of the last 50 entries is 3. Determine the matrix $A \cdot B$.

7. For all real number c determine the rank of the matrix

$$\begin{pmatrix} c & 2 & 3 \\ 21 & 12 & 18 \\ -14 & -8 & -12 \end{pmatrix}$$

8. Let A be an arbitrary $m \times n$ matrix and $b \in \mathbb{R}^m$ an arbitrary vector. True or false:
(a) If the columns of A are linearly independent then the system $Ax = b$ has solution.
(b) If the rows of A are linearly independent then the system $Ax = b$ has solution.