Exploration and Regularization in Reinforcement Learning

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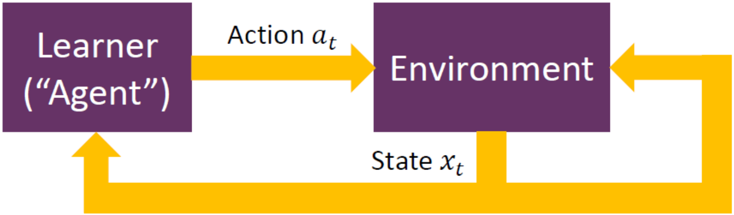
Based on joint work with Anders Jonsson and Vicenç Gómez

Outline

1. MDP basics in 5 minutes
2. Exploration and regularization in RL
3. Entropy-regularized RL
   - Recent trends
   - A unifying theory
   - An algorithmic framework
   - Some results

Markov decision processes



Repeat for $t = 1, 2, \ldots$:

- Learner
  - observes state x_t and plays action a_t
  - obtains reward $r(x_t, a_t)$,
- Environment generates next state $x_{t+1} \sim P(\cdot|x_t, a_t)$.

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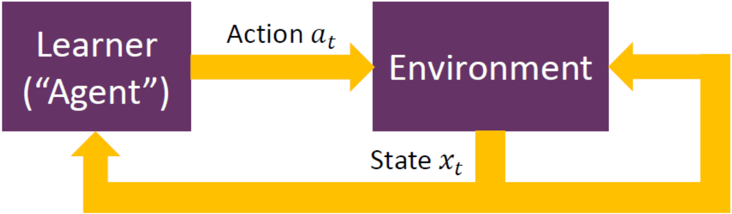


Repeat for $t = 1, 2, \ldots$:

- LEARNER
  - observes state x_t and plays action a_t
  - obtains reward $r(x_t, a_t)$,
- ENVIRONMENT generates next state $x_{t+1} \sim P(\cdot | x_t, a_t)$.

GOAL: gather as much reward as possible

Optimal control in MDPs

A 5-minute summary

- Average-reward criterion:

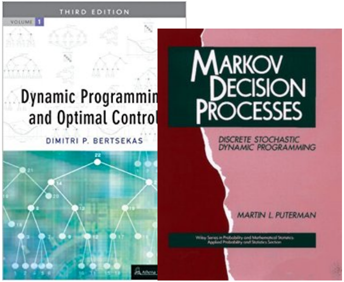
$$\liminf_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} r(x_t, a_t)\right].$$

- Basic fact: enough to consider *stationary policies*

$$\pi(a|x) = \mathbb{P}\left[a_t = a \middle| x_t = x\right].$$

- Under mild assumptions, every π induces stationary distribution μ_π:

$$\mu_\pi(x, a) = \lim_{t\to\infty} \mathbb{P}\left[x_t = x, a_t = a\right].$$



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Notice: average reward of π is linear in μ_π:

$$\lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} r(x_t, a_t)\right]$$

$$= \sum_{x,a} \mu_\pi(x, a) r(x, a)$$

$$= \langle \mu_\pi, r\rangle$$

Optimal control in MDPs

The LP formulation

Primal LP

$$\rho^* = \max_{\mu \in \Delta} \langle \mu, r \rangle$$

$$\Delta = \left\{ \text{distribution } \mu : \sum_b \mu(y, b) = \sum_{x,a} P(y|x, a)\mu(x, a) \quad (\forall y) \right\}$$

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Dual LP

$$\rho^* = \min_{\rho \in \mathbb{R}} \rho$$

$$\text{s.t.} \quad V(x) \geq r(x, a) - \rho + \sum_y P(y|x, a) V(y) \quad (\forall x, a)$$

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Dual "LP" $\equiv$ The Bellman equations

$$V^*(x) = \max_a \left(r(x,a) - \rho^* + \sum_y P(y|x,a)\, V^*(y) \right) \quad (\forall x, a)$$

Reinforcement Learning in MDPs

Reinforcement Learning

$$\approx$$

learning optimal policies in unknown MDPs

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Exactly solving imperfectly known MDPs is foolish!

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SOLUTION:
Regularization!

A recent trend: (Entropy-)Regularized RL

Two popular approaches

Idea 1: Soften the max in the Bellman optimality equations!

$$V^*(x) = \max_a \left(r(x,a) - \rho^* + \sum_y P(y|x,a) V^*(y) \right)$$

A recent trend: (Entropy-)Regularized RL

Two popular approaches

Idea 1: Soften the max in the Bellman optimality equations!

$$V_\eta^*(x) = \frac{1}{\eta} \log \sum_a \exp\left(\eta \left(r(x,a) - \rho_\eta^* + \sum_y P(y|x,a)\, V_\eta^*(y)\right)\right)$$

[Marcus et al., 1997, Ruszczyński, 2010, Ziebart et al., 2010, Ziebart, 2010, Braun et al., 2011, Azar et al., 2012, Rawlik et al., 2012, Fox et al., 2016, Asadi and Littman, 2017, Haarnoja et al., 2017, Schulman et al., 2017, Nachum et al., 2017] . . . and who knows how many more NIPS'17 submissions

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Idea 2: Maximize a regularized objective!

$$\rho(\mu) = \langle \mu, r \rangle$$

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Idea 2: Maximize a regularized objective!

$$\rho_\eta(\mu) = \langle \mu, r \rangle - \frac{1}{\eta} R(\mu)$$

[Peters et al., 2010, Montgomery and Levine, 2016, Schulman et al., 2015, Mnih et al., 2016, O'Donoghue et al., 2017]

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[Marcus ... , Azar
et al., 20 ... 017,
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Numerous open questions:

- are these approaches connected?
- do the derived algorithms converge anywhere?
- does a solution even exist?

A unified framework for entropy-regularized MDPs

Neu, Jonsson and Gómez (2017)

Primal LP

$$\rho^* = \max_{\mu \in \Delta} \langle \mu, r \rangle$$

Dual "LP"

$$V^*(x) = \max_a \left(r(x,a) - \rho^* + \sum_y P(y|x,a) V^*(y) \right) \quad (\forall x, a)$$

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$R(\mu) = ???$

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Conditional entropy regularization

Neu, Jonsson and Gómez (2017)

Theorem

The two convex programs are connected by Lagrangian duality with the choice

$$
\begin{aligned}
R(\mu) &= \sum_{x,a} \mu(x,a) \log \frac{\mu(x,a)}{\sum_b \mu(x,b)} \\
&= \sum_{x,a} \mu(x,a) \log \pi_\mu(a|x)
\end{aligned}
$$

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$$R(\mu) = \sum_{x,a} \mu(x,a) \log \frac{\mu(x,a)}{\sum_b \mu(x,b)} = \sum_{x,a} \mu(x,a) \log \pi_\mu(a|x)$$

Lemma

The conditional entropy $R(\mu)$ is convex in μ and the associated Bregman divergence is

$$D\left(\mu \| \mu'\right) = \sum_{x,a} \mu(x,a) \log \frac{\pi_\mu(a|x)}{\pi_{\mu'}(a|x)} \geq 0.$$

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Primal convex program

Immediate consequences:

- existence & uniqueness results
- well-defined contractive DP operators
- policy gradient theorems...

$$V_\eta^*(x) = \frac{1}{\eta} \log \sum_a \exp\left(\eta\left(r(x,a) - \rho_\eta^* + \sum_y P(y|x,a)\,V_\eta^*(y)\right)\right)$$

A unified algorithmic framework

Neu, Jonsson and Gómez (2017)

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Every algorithm is either Mirror Descent or Dual Averaging!

A unified algorithmic framework

Neu, Jonsson and Gómez (2017)

Every algorithm is either Mirror Descent or Dual Averaging!

- provides a common analytic framework
- ensures convergence
- explains numerous recent algorithms

Example 1:

Trust-region policy optimization $\approx$ Mirror Descent

Neu, Jonsson and Gómez (2017)

Mirror descent

$$\mu_{t+1} = \arg\max_{\mu \in \Delta} \left(\langle \mu, r \rangle - \frac{1}{\eta} D\left(\mu \| \mu_t\right) \right)$$

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Trust-Region Policy Optimization [Schulman et al., 2015]:

$$D_{\mathrm{TRPO}}\left(\mu \| \mu_{\mathrm{old}}\right) = \sum_{x,a} \nu_{\mathrm{old}}(x) \pi_\mu(a|x) \log \frac{\pi_\mu(a|x)}{\pi_{\mathrm{old}}(a|x)}$$

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Corollary

TRPO converges to the optimal policy!

Example 2:

A3C $\approx$ Dual Averaging

Neu, Jonsson and Gómez (2017)

Dual Averaging

$$\mu_{t+1} = \operatorname*{arg\,max}_{\mu \in \Delta} \left(\langle \mu, r \rangle - \frac{1}{\eta_t} R(\mu) \right)$$

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"A3C" [Mnih et al., 2016, O'Donoghue et al., 2017]:

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$$\approx \sum_{x,a} \nu_\mu(x)\pi_\mu(a|x)\log \pi_\mu(a|x) = R(\mu)$$

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Divergence alert!!!

A3C optimizes a non-stationary and non-convex objective!

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Patching A3C:

- O'Donoghue et al. [2017] characterize the stationary points of A3C, but do not show its existence or that A3C would converge to this fixed point

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Neu, Jonsson and Gómez (2017)

Patching A3C:

- O'Donoghue et al. [2017] characterize the stationary points of A3C, but do not show its existence or that A3C would converge to this fixed point
- Our theory provides a closed-form expression for the regularized policy gradient: just replace the advantage function $A^{\pi}(x, a)$ by

$$A_{\eta}^{\pi}(x, a) = r(x, a) - \frac{1}{\eta} \log \pi(a|x) + \sum_{y} P(y|x, a) V_{\eta}^{\pi}(y) - V_{\eta}^{\pi}(x)$$

Other algorithms in our framework

Neu, Jonsson and Gómez (2017)

Mirror Descent:

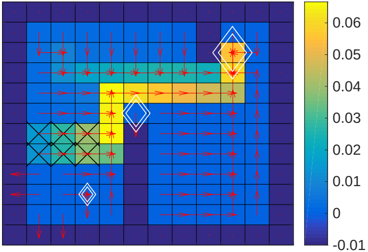
- Dynamic Policy Programming [Azar et al., 2012], Ψ-learning [Rawlik et al., 2012]
- Relative Entropy Policy Search [Peters et al., 2010, Zimin and Neu, 2013, Montgomery and Levine, 2016]

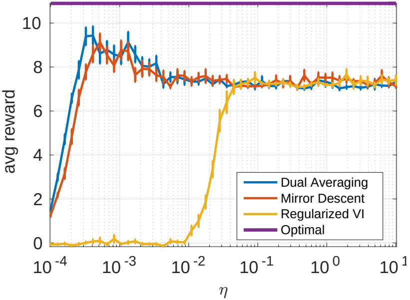
Dual Averaging:

- "MellowMax" RL algorithms of [Asadi and Littman, 2017], G-learning [Fox et al., 2016]
- "Energy-based policy search" [Haarnoja et al., 2017]
- "Path consistency learning" [Nachum et al., 2017]

Experiments

Neu, Jonsson and Gómez (2017)





"Regularization curve":

- η too large: convergence to suboptimal goal $\leftrightarrow$ overfitting
- η too small: policy too close to uniform $\leftrightarrow$ underfitting

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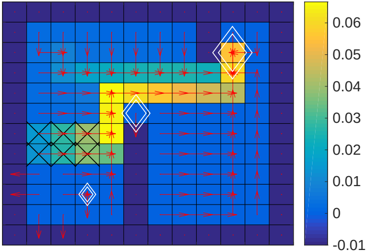

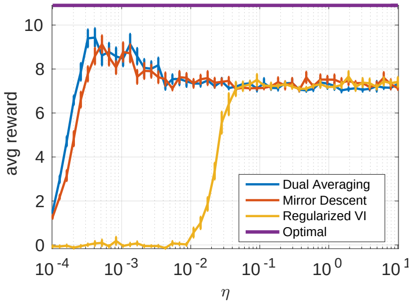

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Dual Averaging perspective seems essential!

- DA theory suggests $\eta_t = t \cdot \eta_0$
- Regularized Value Iteration with constant η is bad

Outlook

Can regularization provide a useful perspective on exploration?

- "Exploration" integrated in the foundations: regularized Bellman equations
- convex optimization framework provides analysis tools and algorithmic templates

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The way towards more effective algorithms?

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