Online Markov Decision Processes under Bandit Feedback

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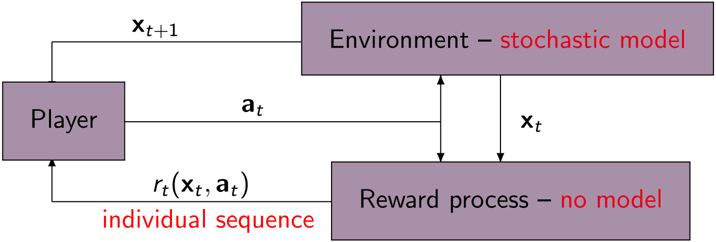
Online Markov Decision Processes



- Goal: minimize regret relative to the best fixed policy

$$\hat{L}_T = \max_\pi R_T^\pi - \hat{R}_T = \max_\pi \mathbb{E}\left[\sum_{t=1}^{T} r_t(\mathbf{x}'_t, \mathbf{a}'_t)\right] - \mathbb{E}\left[\sum_{t=1}^{T} r_t(\mathbf{x}_t, \mathbf{a}_t)\right].$$

- Earlier work:
  - Full information: $\hat{L}_T = O(\sqrt{T})$ (Even-dar et al., 2005).
  - Bandit information: $\hat{L}_T = o(T)$ (Yu et al., 2009).
  - Bandit information for episodic loop-free MDPs: $\hat{L}_T = O(\sqrt{T})$ (Neu et al., 2010).

Online learning with bandit information: the algorithm

- Define unbiased estimates of rewards

$$\hat{\mathbf{r}}_t(x,a) = \begin{cases} \frac{r_t(x,a)}{\mathbf{p}_t^N(x,a|\mathbf{x}_{t-N},\mathbf{a}_{t-N})} & \text{if } (x,a) = (\mathbf{x}_t,\mathbf{a}_t) \\ 0 & \text{otherwise,} \end{cases}$$

  where

$$\mathbf{p}_t^N(x,a|\mathbf{x}_{t-N},\mathbf{a}_{t-N}) = \mathbb{P}\left[\mathbf{x}_t = x, \mathbf{a}_t = a \middle| \mathbf{x}_{1:t-N}, \mathbf{a}_{1:t-N}\right].$$

- Let $\hat{\rho}_t = \mathbb{E}\left[\hat{\mathbf{r}}_t(x,a) \middle| x \sim \mu^{\boldsymbol{\pi}_t}, a \sim \boldsymbol{\pi}_t\right]$ and $\hat{\mathbf{q}}_t$ be the solution to the Bellman equations

$$\hat{\mathbf{q}}_t(x,a) = \hat{\mathbf{r}}_t(x,a) - \hat{\rho}_t + \sum_{x',a'} P(x'|x,a)\boldsymbol{\pi}_t(a'|x')\hat{\mathbf{q}}_t(x',a').$$

- Feed an instance of **Exp3** with the computed values of $\hat{\mathbf{q}}_t(x,a)$ in each state x.

Online learning in MDPs with bandit information

- Assume:
  - General MDP with cycles.
  - Every policy π induces a stationary distribution μ^π over the states.
  - Every policy mixes fast (with mixing time τ).
  - $\mu^\pi(x) \geq \alpha > 0$ for all π.
- Result: sublinear regret relative to the best fixed policy

$$\hat{L}_T = \mathcal{O}\left(\tau T^{2/3}\left(\frac{|\mathcal{A}| \log(|\mathcal{A}|) \log(T)}{\alpha}\right)^{1/3}\right).$$

- Proof idea:

$$\hat{L}_T = \underbrace{\left(R_T^\pi - \sum_{t=1}^T \rho_t^\pi\right)}_{\leq 2\tau+2} + \underbrace{\left(\sum_{t=1}^T \rho_t^\pi - \sum_{t=1}^T \rho_t^{\boldsymbol{\pi}_t}\right)}_{\mathcal{O}(T^{2/3})} + \underbrace{\left(\sum_{t=1}^T \rho_t^{\boldsymbol{\pi}_t} - \hat{R}_T\right)}_{\mathcal{O}(T^{2/3})}$$

See you at poster 95!